Arbitrage and Financial Decision Making

How Do We Evaluate a Decision

- Identify costs and benefits
- Comparing costs and benefits in common terms
  - in terms of cash today
- Does the cash value today of its benefits exceed the cash value today of its costs?

Lecture Outline

- Introduction to Present Value
- No arbitrage opportunities in competitive markets
- The law of one price
- The price of risk

Time Value of Money

- Consider an investment opportunity with the following certain cash flows.
  - Cost: $9,500 today
  - Benefit: $10,500 in one year
  - Net Value: $1,000
- **A dollar today is worth more than a dollar in one year**
- The difference in value between money today and money in the future is called the time value of money.

The Interest Rate: An Exchange Rate Across Time

- Suppose the current annual interest rate is 7%. By investing or borrowing at this rate, we can exchange \$1.07 in one year for each \$1 today.
- Risk–Free Interest Rate r_f
- Interest Rate Factor = $1 + r_f$
- Discount Factor = $1 / (1 + r_f)$

Example

Comparing Costs at Different Points in Time

Problem

The cost of rebuilding the San Francisco Bay Bridge to make it earthquake-safe was approximately \$3 billion in 2004. At the time, engineers estimated that if the project were delayed to 2005, the cost would rise by 10%. If the interest rate was 2%, what was the cost of a delay in terms of dollars in 2004?

Example

Solution

If the project were delayed, it would cost \$3 billion $\times$ (1.10) = \$3.3 billion in 2005. To compare this amount to the cost of \$3 billion in 2004, we must convert it using the interest rate of 2%:

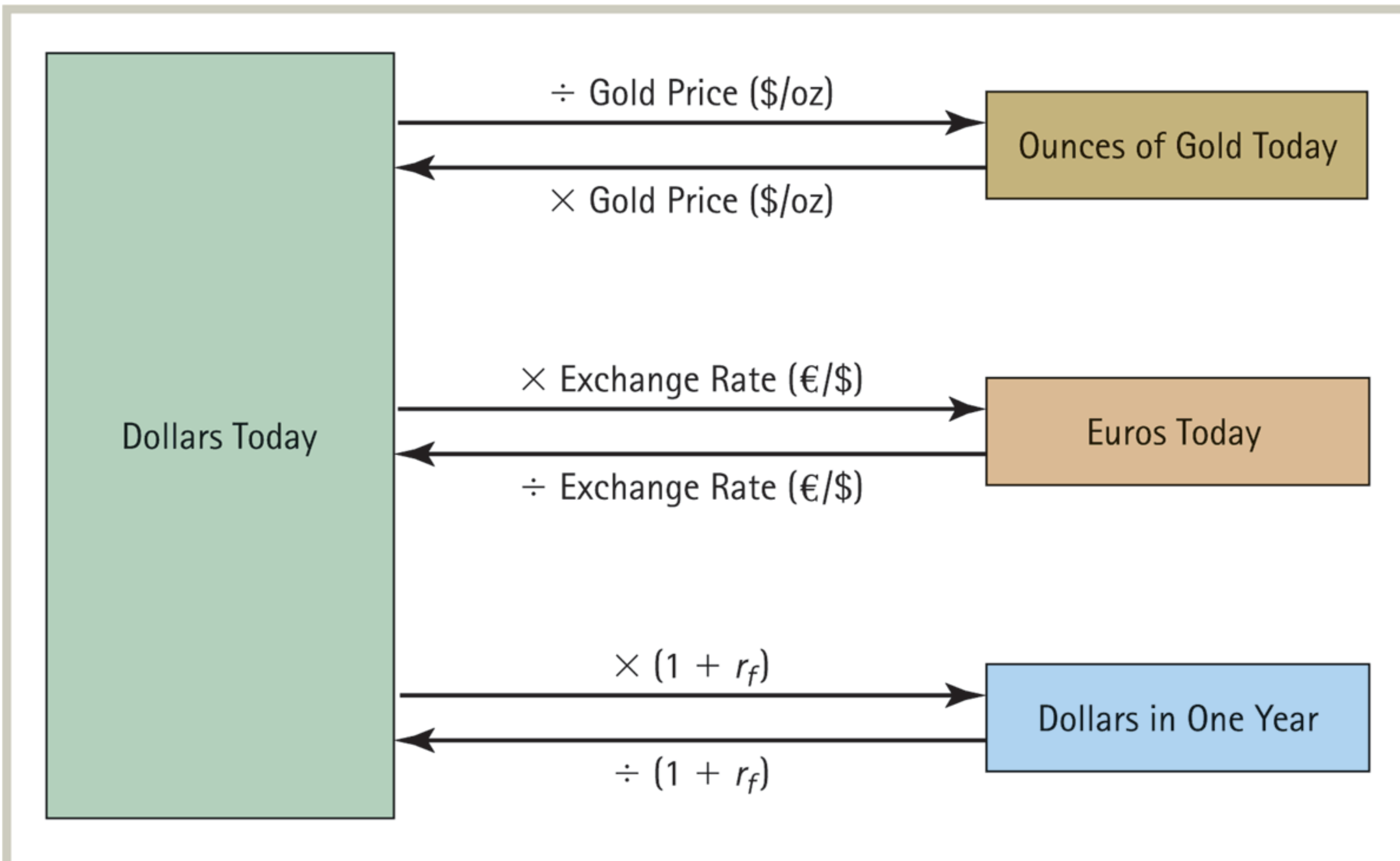
$$\$3.3 \text{ billion in } 2005 \div (\$1.02 \text{ in } 2005 / \$ \text{ in } 2004) = \$3.235 \text{ billion in } 2004$$

Therefore, the cost of a delay of one year was

$$\$3.235 \text{ billion} - \$3 \text{ billion} = \$235 \text{ million in } 2004$$

That is, delaying the project for one year was equivalent to giving up \$235 million in cash.

The Interest Rate: An Exchange Rate Across Time



Net Present Value

- The **net present value (NPV)** of a project or investment is the difference between the present value of its benefits and the present value of its costs.

$$NPV = PV(\text{All project cash flows})$$

$$NPV = PV(\text{Benefits}) - PV(\text{Costs})$$

Example

- Suppose an investment that promises to pay \$10,500 in one year is offered for sale for \$9,500. Your interest rate is 5%. If you invest in this project, the NPV of your investment is

$$NPV = -9500 + \frac{10500}{1+0.05} = 500$$

The NPV Decision Rule

- Accepting or Rejecting a Project
  - Accept those projects with positive NPV because accepting them is equivalent to receiving their NPV in cash today.
  - Reject those projects with negative NPV because accepting them would reduce the wealth of investors.

Choosing Among Projects

TABLE 3.1 **Cash Flows of Three Possible Projects**

Project	Cash Flow Today ($)	Cash Flow in One Year ($)
A	42	42
B	−20	144
C	−100	225

TABLE 3.2 **Computing the NPV of Each Project**

Project	Cash Flow Today ($)	PV of Cash Flow in One Year ($)	NPV ($ Today)
A	42	$42 \div 1.20 = 35$	$42 + 35 = 77$
B	−20	$144 \div 1.20 = 120$	$-20 + 120 = 100$
C	−100	$225 \div 1.20 = 187.5$	$-100 + 187.5 = 87.5$

Choosing Among Projects

- All three projects have positive NPV, and we would accept all three if possible.
- If we must choose only one project, Project B has the highest NPV and therefore is the best choice.

TABLE 3.1 Cash Flows of Three Possible Projects

Project	Cash Flow Today ($)	Cash Flow in One Year ($)
A	42	42
B	−20	144
C	−100	225

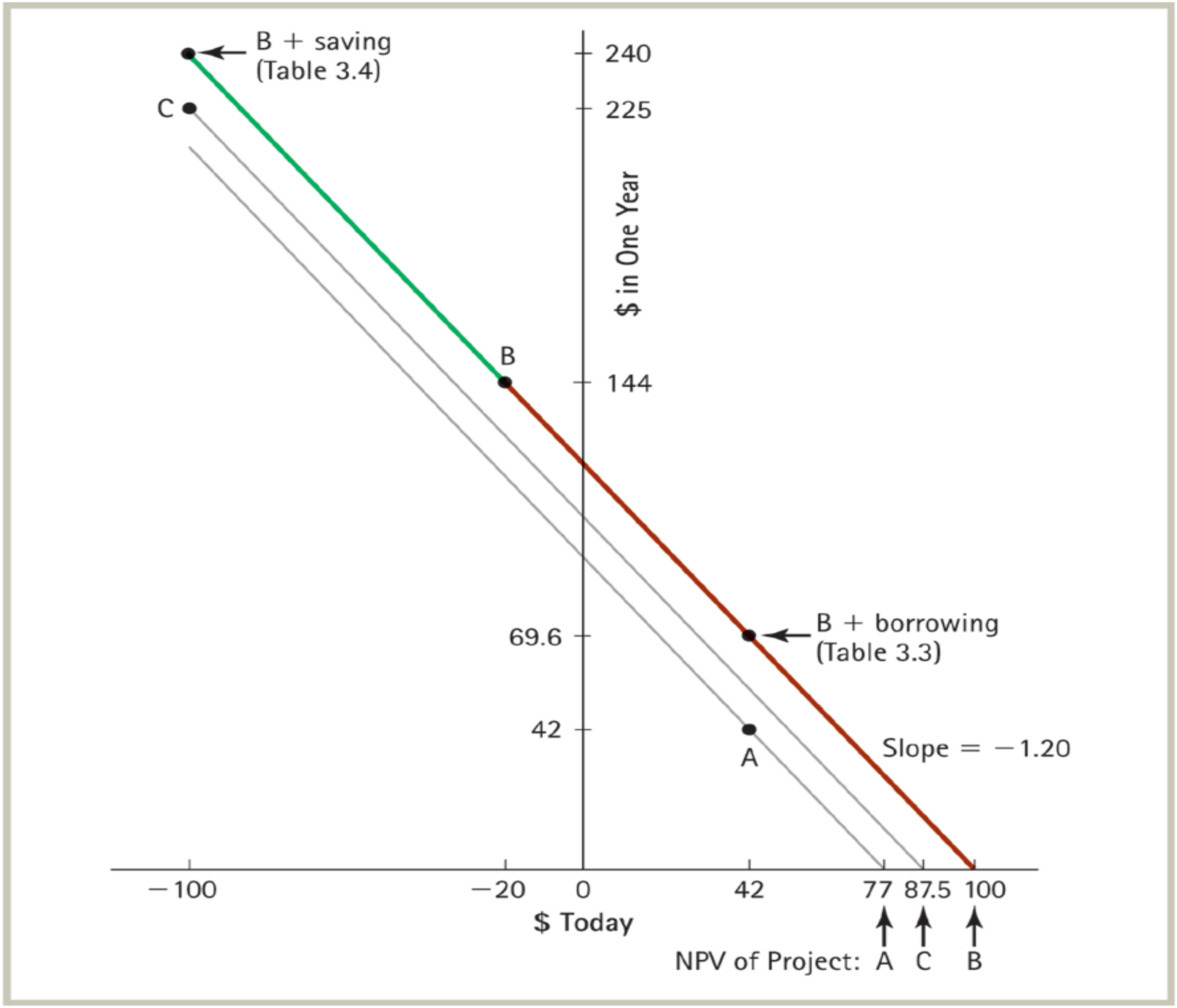
- Although Project B has the highest NPV, what if we do not want to spend the $20 for the cash outlay? Would Project A be a better choice? Should this affect our choice of projects?
- NO! As long as we are able to borrow and lend at the risk-free interest rate, Project B is superior whatever our preferences regarding the timing of the cash flows.

TABLE 3.3 **Cash Flows from Combining Project B with Borrowing**

	Cash Flow Today ($)	Cash Flow in One Year ($)
Project B	−20	144
Borrow	62	$-62 \times (1.20) = -74.4$
Total	42	69.6

TABLE 3.4 **Cash Flows from Combining Project B with Saving**

	Cash Flow Today ($)	Cash Flow in One Year ($)
Project B	−20	144
Save	−80	$80 \times (1.20) = 96$
Total	−100	240



B + saving
(Table 3.4)
C
240
225
$ in One Year
B
144
69.6
42
A
B + borrowing
(Table 3.3)
Slope = −1.20
−100
−20
0
42
77
87.5
100
$ Today
NPV of Project: A C B

The separation theorem

- **The investment decision is independent of consumption/savings decision**

- Regardless of our preferences for cash today versus cash in the future, we should always maximize NPV first. We can then borrow or lend to shift cash flows through time and find our most preferred pattern of cash flows.

Arbitrage and The Law of One Price

- Arbitrage
  - The opportunity to buy and sell equivalent goods in different markets to exploit a price difference. Such opportunity arises when it is possible to make a profit without taking any risk or incurring any cost.
- The Law of One Price
  - If equivalent investment opportunities trade simultaneously in different competitive markets, then they must trade for the same price in both markets. Arbitrage will quickly eliminate any price difference for equivalent investment opportunities.

Valuing a Security

- Assume a security promises a risk-free payment of $1000 in one year. If the risk-free interest rate is 5%, what can we conclude about the price of this bond in a normal market?

$$\begin{aligned} PV(\$1000 \text{ in one year}) &= (\$1000 \text{ in one year}) \div (1.05 \text{ \$ in one year / \$ today}) \\ &= \$952.38 \text{ today} \end{aligned}$$

Price(Bond) = $952.38

Valuing a Security

- Assume the price is $940.

TABLE 3.5 **Net Cash Flows from Buying the Bond and Borrowing**

	Today ($)	In One Year ($)
Buy the bond	−940.00	+1000.00
Borrow from the bank	+952.38	−1000.00
Net cash flow	+12.38	0.00

- Assume the price is $960.

TABLE 3.6 **Net Cash Flows from Selling the Bond and Investing**

	Today ($)	In One Year ($)
Sell the bond	+960.00	−1000.00
Invest at the bank	−952.38	+1000.00
Net cash flow	+7.62	0.00

Valuing a Security

- Unless the price of the security equals the present value of the security's cash flows, an arbitrage opportunity will appear.
- No Arbitrage Price of a Security

$$\text{Price(Security)} = PV(\text{All cash flows paid by the security})$$

Valuing a Security

- If we know the price of a risk-free bond, we can use

$$\text{Price(Security)} = PV(\text{All cash flows paid by the securi}$$

to determine what the risk-free interest rate must be if there are no arbitrage opportunities.

Example

- Suppose a risk-free bond that pays $1000 in one year is currently trading with a competitive market price of $929.80 today.

$$\$929.80 \text{ today} = (\$1000 \text{ in one year}) \div (1 + r_f \ \$ \text{ in one year} / \$ \text{ today})$$

$$1 + r_f = \frac{\$1000 \text{ in one year}}{\$929.80 \text{ today}} = 1.0755 \ \$ \text{ in one year} / \$ \text{ today}$$

- The risk-free interest rate must be 7.55%.

Valuing a Portfolio

- Consider two securities, A and B. Suppose a third security, C, has the same cash flows as A and B combined.

$$\text{Price}(C) = \text{Price}(A + B) = \text{Price}(A) + \text{Price}(B)$$

The Law of One Price

Value Additivity

Valuing a Portfolio

TABLE 3.8 **Determining the Market Price of Security A (cash flows in $)**

Security	Market Price Today	Cash Flow in One Year	
		Weak Economy	Strong Economy
Risk-free bond	769	800	800
Security A	?	0	600
Market index	1000	800	1400

The Price of Risk

- Risky Versus Risk-free Cash Flows

TABLE 3.7 **Cash Flows and Market Prices (in $) of a Risk-Free Bond and an Investment in the Market Portfolio**

Security	Market Price Today	Cash Flow in One Year: Weak Economy	Cash Flow in One Year: Strong Economy
Risk-free bond	1058	1100	1100
Market index	1000	800	1400

$$\begin{aligned}\text{Price(Risk - free Bond)} &= \text{PV(Cash Flows)}\\ &= (\$1100 \text{ in one year}) \div (1.04 \text{ \$ in one year / \$ today})\\ &= \$1058 \text{ today}\end{aligned}$$

The Price of Risk

- Risk Aversion
  - Investors prefer to have a safe income rather than a risky one of the same average amount.
- Risk Premium
  - The additional return that investors expect to earn to compensate them for a security's risk.

TABLE 3.7 **Cash Flows and Market Prices (in $) of a Risk-Free Bond and an Investment in the Market Portfolio**

Security	Market Price Today	Cash Flow in One Year: Weak Economy	Cash Flow in One Year: Strong Economy
Risk-free bond	1058	1100	1100
Market index	1000	800	1400

- Market return if the economy is strong
  - (1400 – 1000) / 1000 = 40%
- Market return if the economy is weak
  - (800 – 1000) / 1000 = –20%
- Expected market return
  - ½ (40%) + ½ (–20%) = 10%
- Market index’s risk premium=10%-4%=6%

The Price of Risk

- Risk Premiums Depend on Risk

- If an investment has much more variable returns, it must pay investors a higher risk premium.

The Price of Risk

- Risk Is Relative to the Overall Market
- A security's risk premium will be higher the more its returns tend to vary with the overall economy and the market index.
- If the security's returns vary in the opposite direction of the market index, it offers insurance and will have a negative risk premium.

Security	Market Price Today	Cash Flow in One Year	
		Weak Economy	Strong Economy
Risk-free bond	769	800	800
Security A	?	0	600
Market index	1000	800	1400

Security	Market Price Today	Cash Flow in One Year	
		Weak Economy	Strong Economy
Market index	1000	800	1400
Security B	?	600	0
Risk-free bond	1346	1400	1400

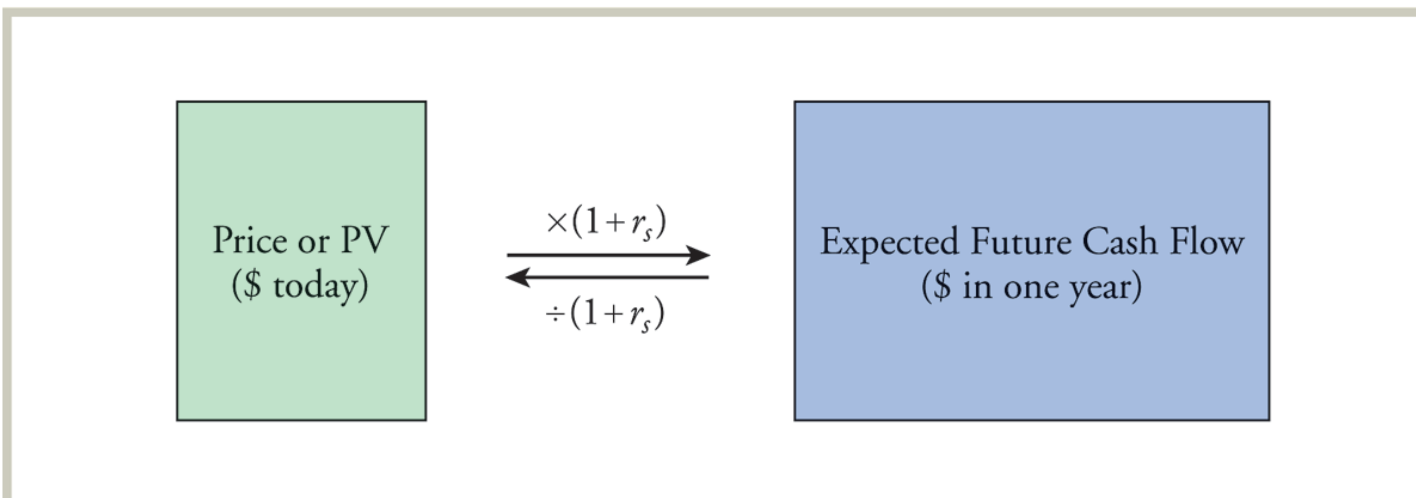
TABLE 3.9 Risk and Risk Premiums for Different Securities

Security	Returns: Weak Economy	Returns: Strong Economy	Difference in Returns	Risk Premium
Risk-free bond	4%	4%	0%	0%
Market index	−20%	40%	60%	6%
Security A	−100%	160%	260%	26%
Security B	73%	−100%	−173%	−17.3%

The Price of Risk

- Computing prices using a discount rate r_s that includes a risk premium appropriate for the investment's risk:

$$r_s = r_f + (\text{risk premium for investment } s)$$



Example

Using the Risk Premium to Compute a Price

Problem

Consider a risky bond with a cash flow of $1100 when the economy is strong and $1000 when the economy is weak. Suppose a 1% risk premium is appropriate for this bond. If the risk-free interest rate is 4%, what is the price of the bond today?

Example

Solution

From Eq. 3.7, the appropriate discount rate for the bond is

$$r_b = r_f + \text{(Risk Premium for the Bond)} = 4\% + 1\% = 5\%$$

The expected cash flow of the bond is $\frac{1}{2}(\$1100) + \frac{1}{2}(\$1000) = \$1050$ in one year. Thus, the price of the bond today is

$$\begin{aligned} \text{Bond Price} &= \text{(Average cash flow in one year)} \div (1 + r_b \text{ \$ in one year / \$ today}) \\ &= (\$1050 \text{ in one year}) \div (1.05 \text{ \$ in one year / \$ today}) \\ &= \$1000 \text{ today} \end{aligned}$$

Given this price, the bond's return is 10% when the economy is strong, and 0% when the economy is weak. (Note that the difference in the returns is 10%, which is 1/6 as variable as the market index; see Table 3.9. Correspondingly, the risk premium of the bond is 1/6 that of the market index as well.)

Quick Quiz

- Explain time value of money
- Why there is no arbitrary opportunities in competitive markets?
- What is the law of one price?