Interest Rate

Annual Percentage Rate

- This is the annual rate that is quoted by law
- By definition APR = period rate times the number of periods per year
- Consequently, to get the period rate we rearrange the APR equation:
  - Period rate = APR / number of periods per year

Computing APRs

- What is the APR if the monthly rate is .5%?
  - .5(12) = 6%
- What is the APR if the semiannual rate is .5%?
  - .5(2) = 1%
- What is the monthly rate if the APR is 12% with monthly compounding?
  - 12 / 12 = 1%

Effective Annual Rate (EAR)

- This is the actual rate paid (or received) after accounting for compounding that occurs during the year
- If you want to compare two alternative investments with different compounding periods you need to compute the EAR and use that for comparison.

EAR - Formula

$$EAR = \left[1 + \frac{APR}{m}\right]^{m} - 1$$

Remember that the APR is the quoted rate

Computing EARs - Example

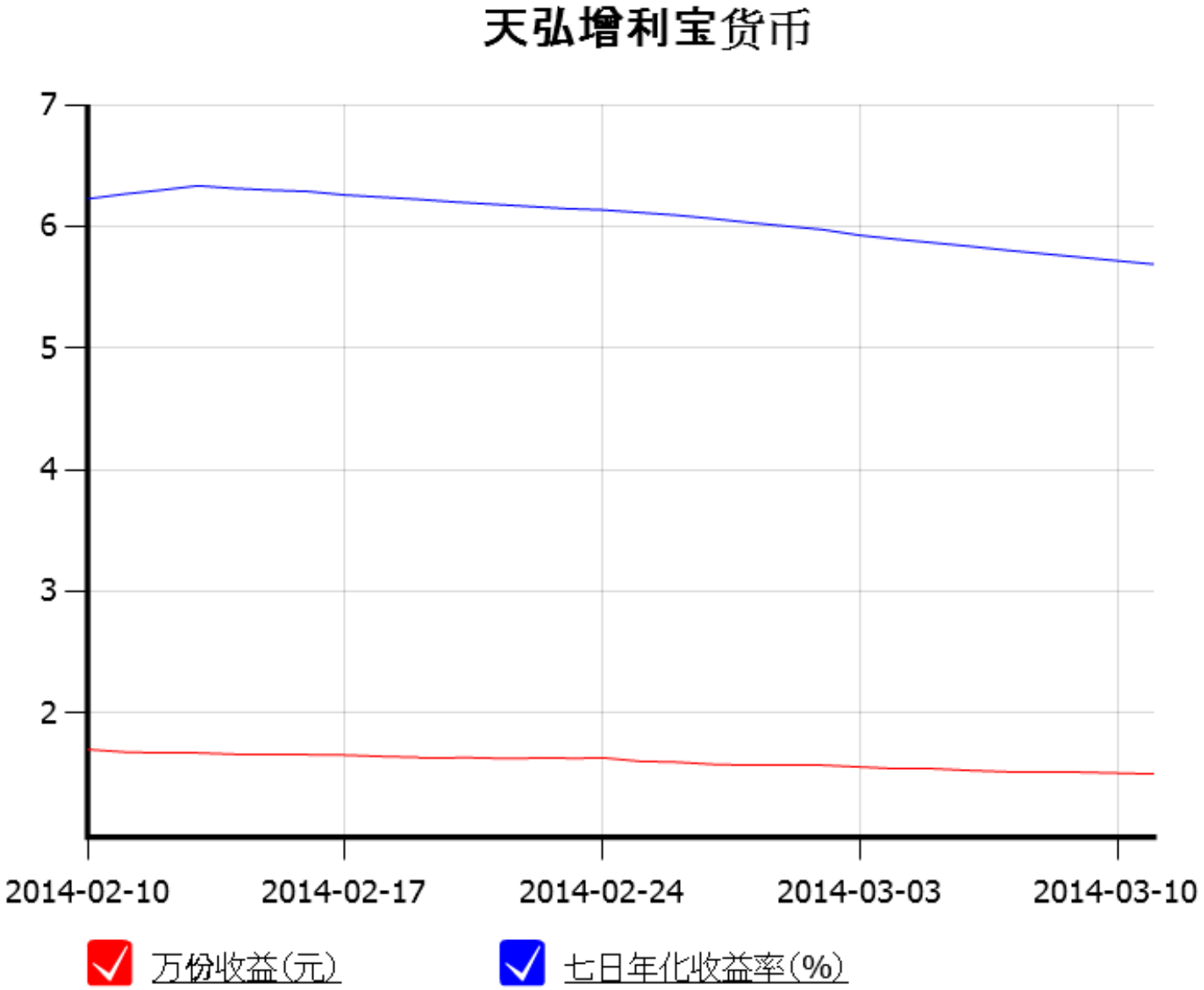
- Suppose you can earn 1% per month on $1 invested today.
  - What is the APR? 1(12) = 12%
  - How much are you effectively earning?
    - FV = $1(1.01)^{12}$ = 1.1268
    - Rate = (1.1268 – 1) / 1 = .1268 = 12.68%
- Suppose if you put it in another account, you earn 3% per quarter.
  - What is the APR? 3(4) = 12%
  - How much are you effectively earning?
    - FV = $1(1.03)^{4}$ = 1.1255
    - Rate = (1.1255 – 1) / 1 = .1255 = 12.55%

Quick Quiz

投资回报

基金名称	每万份收益	七日年化收益率
增利宝	1.4965	5.690%

- What is Alipay's EAR?


天弘增利宝货币
7
6
5
4
3
2
2014-02-10
2014-02-17
2014-02-24
2014-03-03
2014-03-10
万份收益(元)
七日年化收益率(%)

Why is it important to consider effective rates of return?

- An investment with monthly payments is different from one with quarterly payments. Must put each return on an EAR% basis to compare rates of return. Must use EAR% for comparisons. See following values of EAR% rates at various compounding levels.

EAR_{ANNUAL}	10.00%
$EAR_{QUARTERLY}$	10.38%
$EAR_{MONTHLY}$	10.47%
$EAR_{DAILY\ (365)}$	10.52%

Decisions, Decisions II

- You are looking at two savings accounts. One pays 5.25%, with daily compounding. The other pays 5.3% with semiannual compounding. Which account should you use?
  - First account:
    - $EAR = (1 + .0525/365)^{365} - 1 = 5.39\%$
  - Second account:
    - $EAR = (1 + .053/2)^{2} - 1 = 5.37\%$
- Which account should you choose and why?

Decisions, Decisions II Continued

- Let's verify the choice. Suppose you invest $100 in each account. How much will you have in each account in one year?
  - First Account:
    - Daily rate = .0525 / 365 = .00014383562
    - 365 N; 5.25 / 365 = .014383562 I/Y; 100 PV; CPT FV = 105.39
  - Second Account:
    - Semiannual rate = .0539 / 2 = .0265
    - 2 N; 5.3 / 2 = 2.65 I/Y; 100 PV; CPT FV = 105.37
- You have more money in the first account.

Computing APRs from EARs

- If you have an effective rate, how can you compute the APR? Rearrange the EAR equation and you get:

$$APR = m\left[(1 + EAR)^{1/m} - 1\right]$$

APR - Example

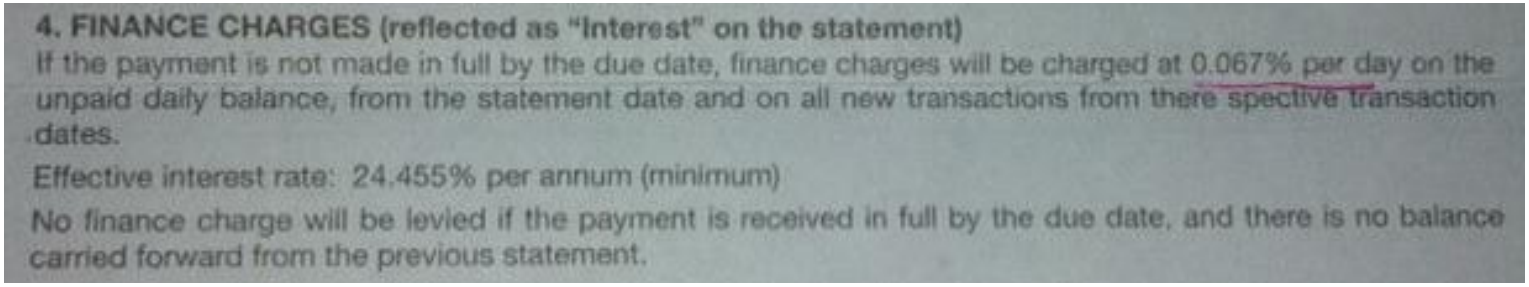
- Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

$$APR = 12\left[(1+.12)^{1/12} - 1\right] = .1138655152$$

or 11.39%

My Credit Card Charge

(a) Standard Chartered Bank



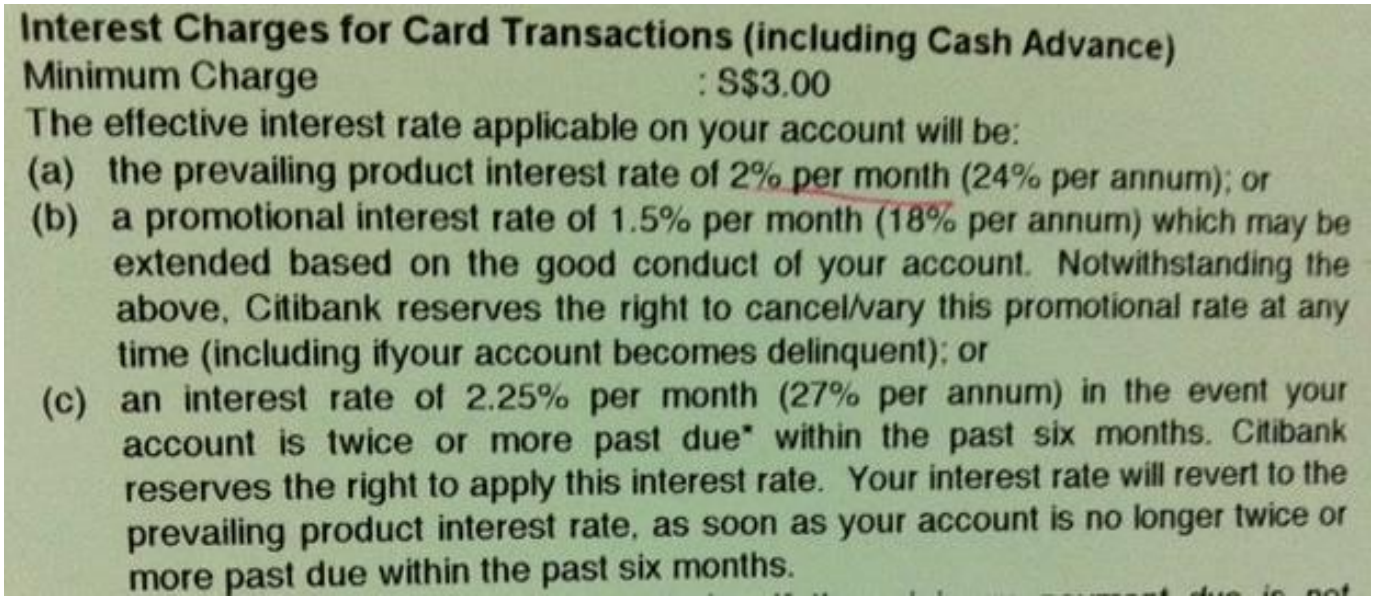
4. FINANCE CHARGES (reflected as "Interest" on the statement)

If the payment is not made in full by the due date, finance charges will be charged at 0.067% per day on the unpaid daily balance, from the statement date and on all new transactions from there spective transaction dates.

Effective interest rate: 24.455% per annum (minimum)

No finance charge will be levied if the payment is received in full by the due date, and there is no balance carried forward from the previous statement.

(b) Citibank



Interest Charges for Card Transactions (including Cash Advance)

Minimum Charge : S$3.00

The effective interest rate applicable on your account will be:

(a) the prevailing product interest rate of 2% per month (24% per annum); or

(b) a promotional interest rate of 1.5% per month (18% per annum) which may be extended based on the good conduct of your account. Notwithstanding the above, Citibank reserves the right to cancel/vary this promotional rate at any time (including ifyour account becomes delinquent); or

(c) an interest rate of 2.25% per month (27% per annum) in the event your account is twice or more past due* within the past six months. Citibank reserves the right to apply this interest rate. Your interest rate will revert to the prevailing product interest rate, as soon as your account is no longer twice or more past due within the past six months.

Interest Rates and Inflation

- So far, we have worked with cash flows in dollars, or nominal cash flows. But, we care about their real cash flows, i.e. how much goods our dollars can buy. Inflation erodes the purchasing power of dollars in the future.
- Mathematically, $1 + R = \frac{1 + r}{1 + i^e}$

  where i^e is the expected inflation rate, R is the real interest rate, and r is the nominal (or market) rate of interest

Continuous Compounding

- The general formula for the future value of an investment compounded continuously over many periods can be written as:

$$FV = C_0 \times e^{rT}$$

Where

C_0 is cash flow at date 0,

r is the stated annual interest rate,

T is the number of periods over which the cash is invested, and

e is a transcendental number approximately equal to 2.718.

The Determinants of Interest Rates

- Inflation and Real Versus Nominal Rates
  - **Nominal Interest Rate:** The rates quoted by financial institutions and used for discounting or compounding cash flows
  - **Real Interest Rate:** The rate of growth of your purchasing power, after adjusting for inflation

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$$\text{Growth in Purchasing Power}$$

$$= 1 + r_r$$

$$= \frac{1 + r}{1 + i}$$

$$= \frac{\text{Growth of Money}}{\text{Growth of Prices}}$$

- The Real Interest Rate

$$r_r = \frac{r - i}{1 + i} \approx r - i$$

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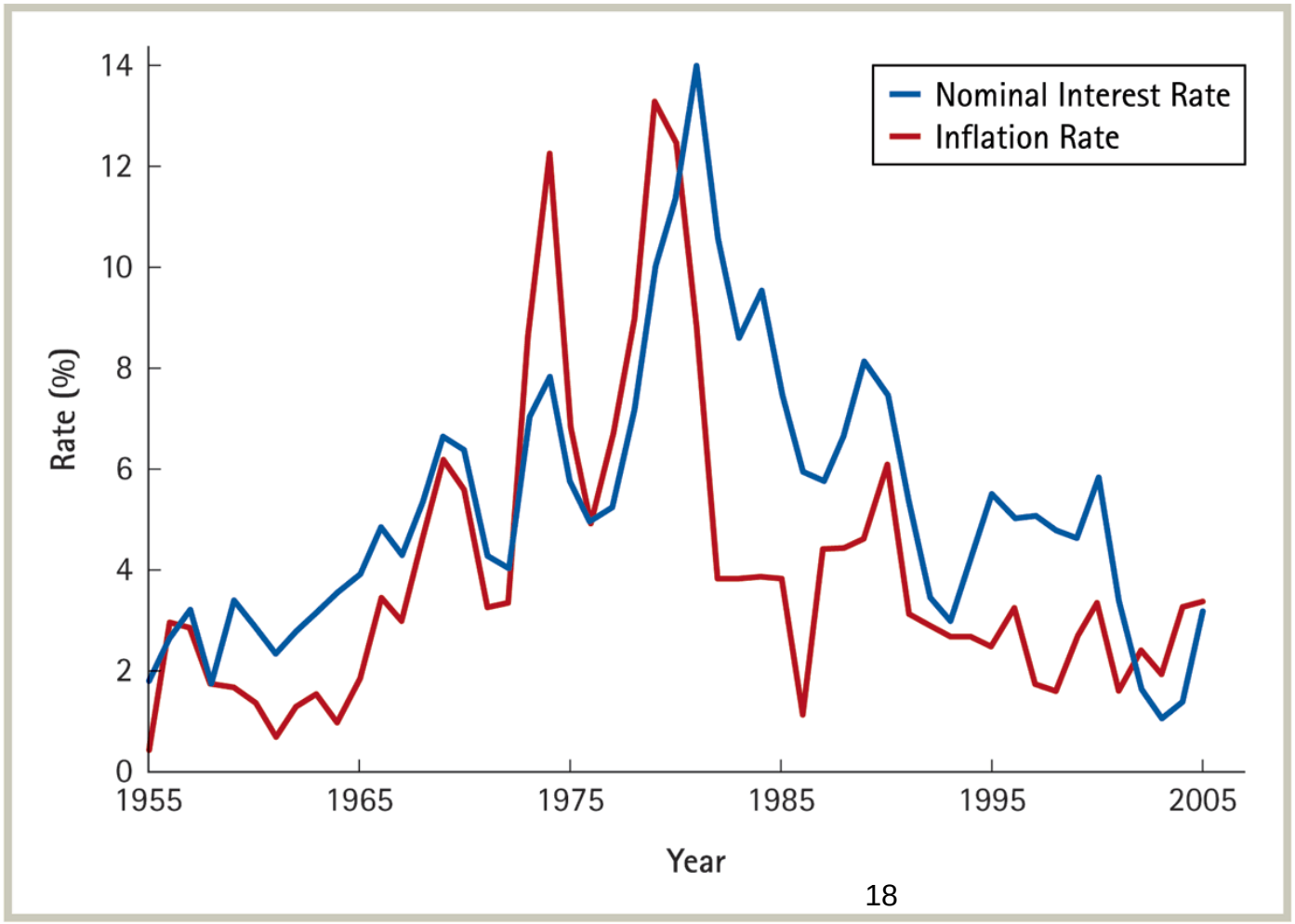
U.S. Interest Rates and Inflation Rates, 1955–2005



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The Yield Curve and Discount Rates

- **Term Structure:** The relationship between the investment term and the interest rate
- **Yield Curve:** A graph of the term structure

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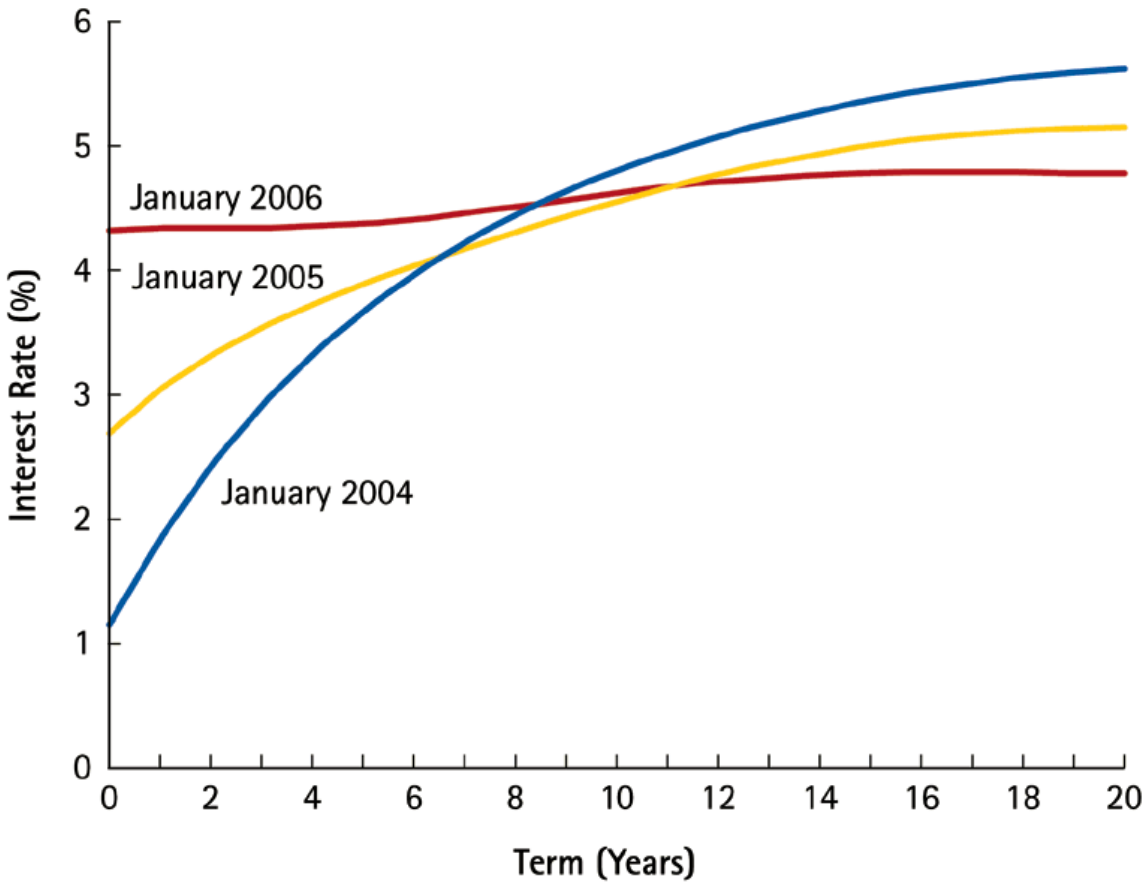
Term Structure of Risk-Free U.S. Interest Rates, January 2004, 2005, and 2006

Term (years)	Date Jan. 2004	Jan. 2005	Jan. 2006
1	1.15%	2.69%	4.32%
2	1.87%	3.06%	4.34%
3	2.48%	3.34%	4.34%
4	2.98%	3.57%	4.34%
5	3.40%	3.76%	4.36%
6	3.75%	3.93%	4.38%
7	4.05%	4.08%	4.42%
8	4.31%	4.22%	4.48%
9	4.53%	4.36%	4.53%
10	4.72%	4.49%	4.59%
11	4.88%	4.61%	4.65%
12	5.02%	4.73%	4.70%
13	5.15%	4.83%	4.73%
14	5.25%	4.91%	4.76%
15	5.35%	4.99%	4.78%
16	5.43%	5.05%	4.79%
17	5.49%	5.09%	4.79%
18	5.55%	5.12%	4.79%
19	5.59%	5.14%	4.78%
20	5.62%	5.15%	4.78%



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- The term structure can be used to compute the present and future values of a risk-free cash flow over different investment horizons.

$$PV = \frac{C_n}{(1 + r_n)^n}$$

  - Present Value of a Cash Flow Stream Using a Term Structure of Discount Rates

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \cdots + \frac{C_N}{(1+r_N)^N} = \sum_{n=1}^{N} \frac{C_N}{(1+r_n)^n}$$

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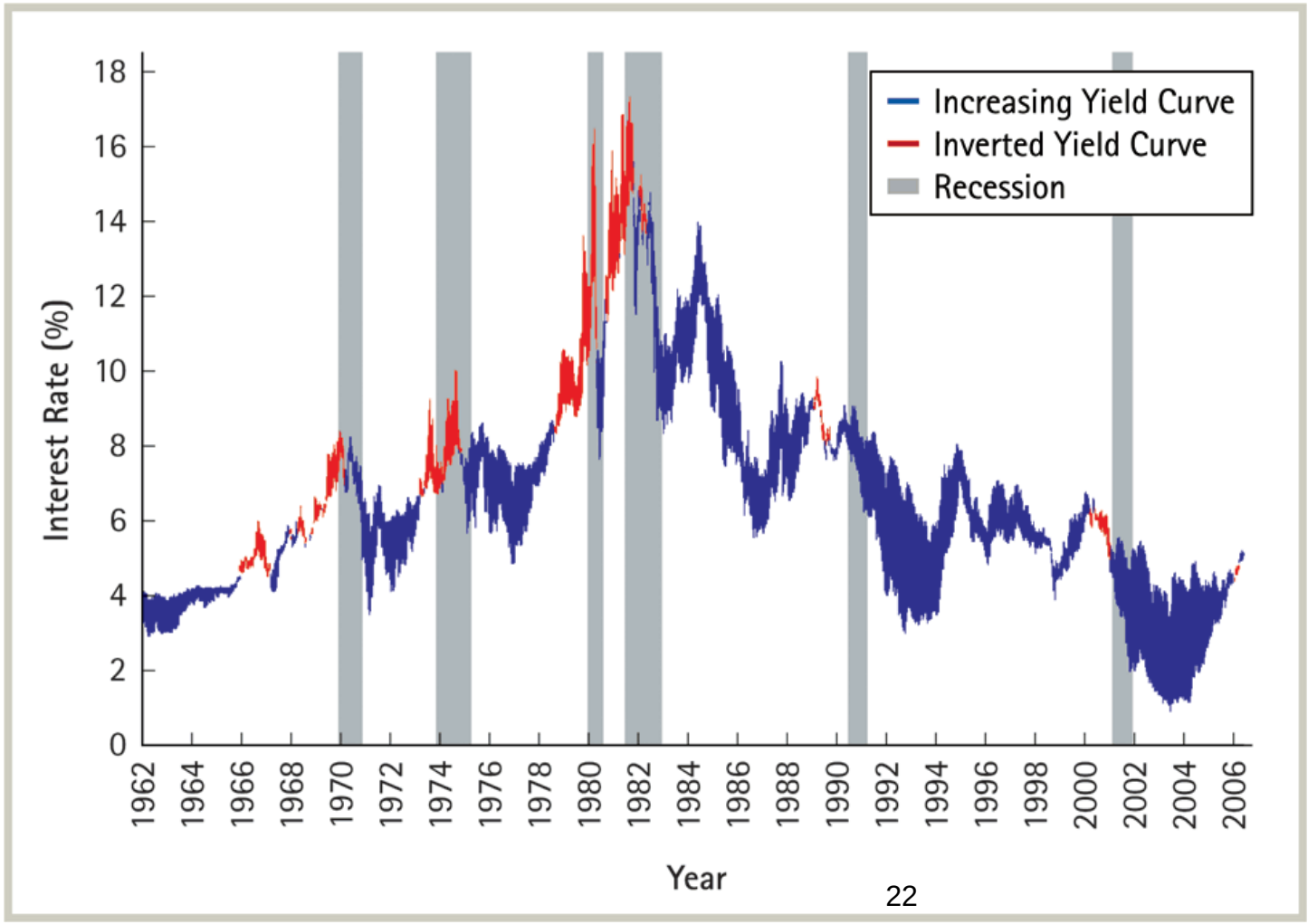
Short-Term Versus Long-Term U.S. Interest Rates and Recessions



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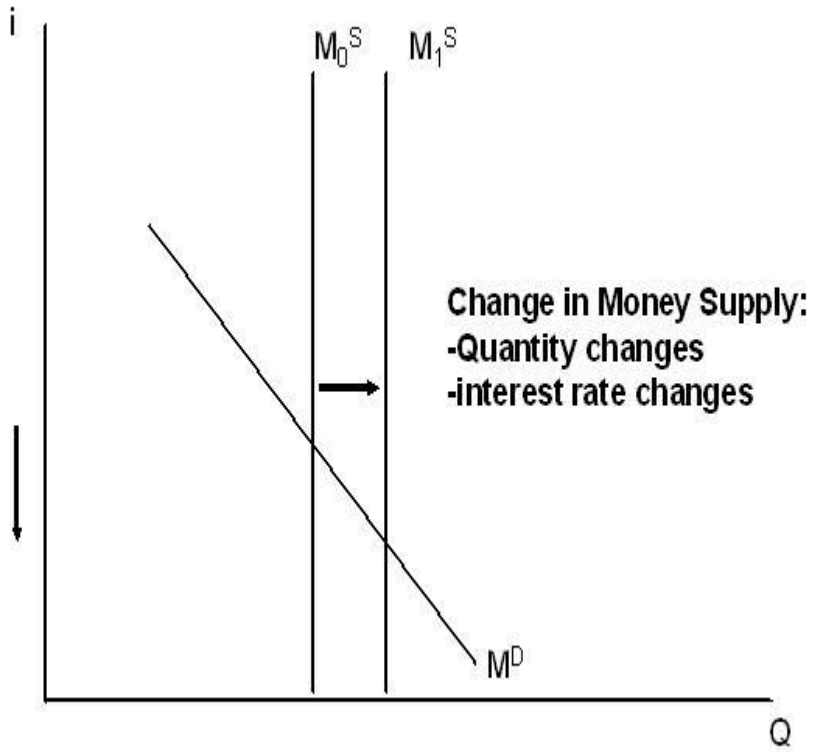
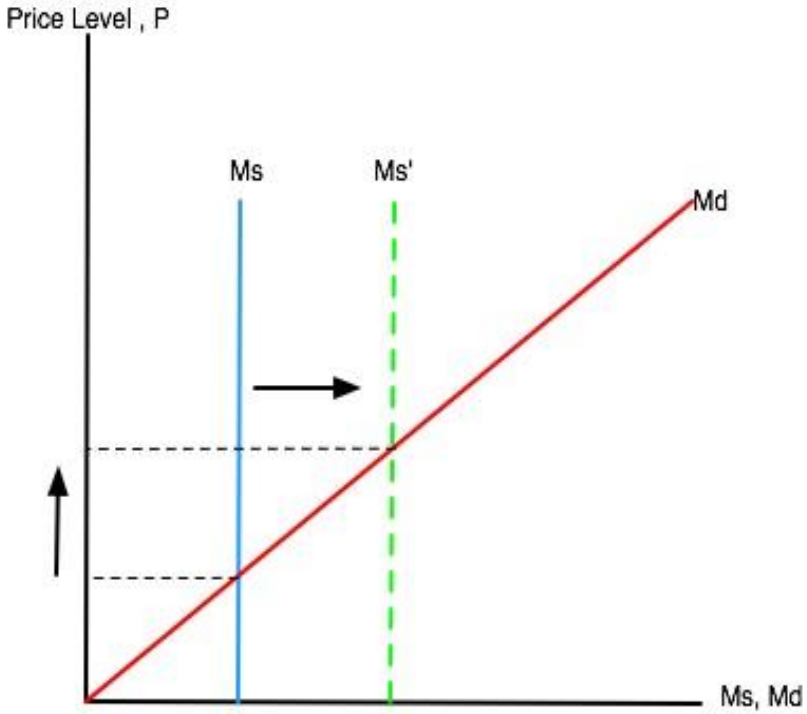


Change in Money Supply:
-Quantity changes
-interest rate changes



Quick Quiz

You borrow $200,000 to buy a house. You have a 9% 30-year fixed-rate mortgage. The mortgage requires monthly payment.

1. What is the monthly payment? ($1,609.25)
2. How much of the principal is still owed after 5 years? ($191,760.84)
3. What happens if you pay more than $1,609/month, say, $1,700/month? How much faster do you pay off the loan? (73.6 mths)
4. After 5 years, mortgage rate falls to 8% (for a 25-year loan). Should you refinance your mortgage? Assume that the fee is $1,200. (Yes)