Interest Rates and Bond Valuation

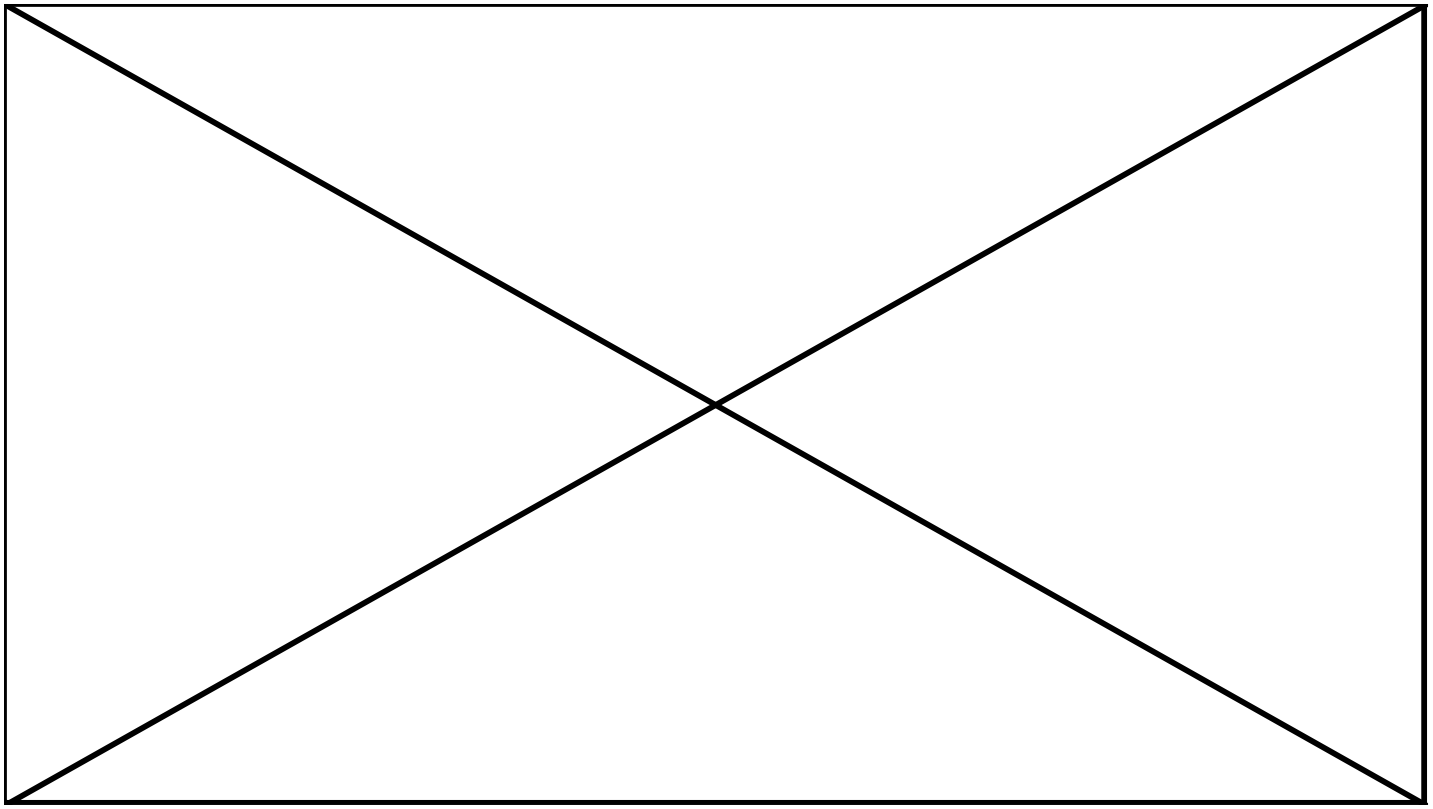
Lecture Outline

- Bonds and Bond Valuation
- Determinants of Bond Yields
- Bond Price Changes
- Bond Ratings
- More on Bond Features
- Term Structure of Interest

What is a bond?

- A long-term debt instrument in which a borrower agrees to make payments of principal and interest, on specific dates, to the holders of the bond.

Introducing bond

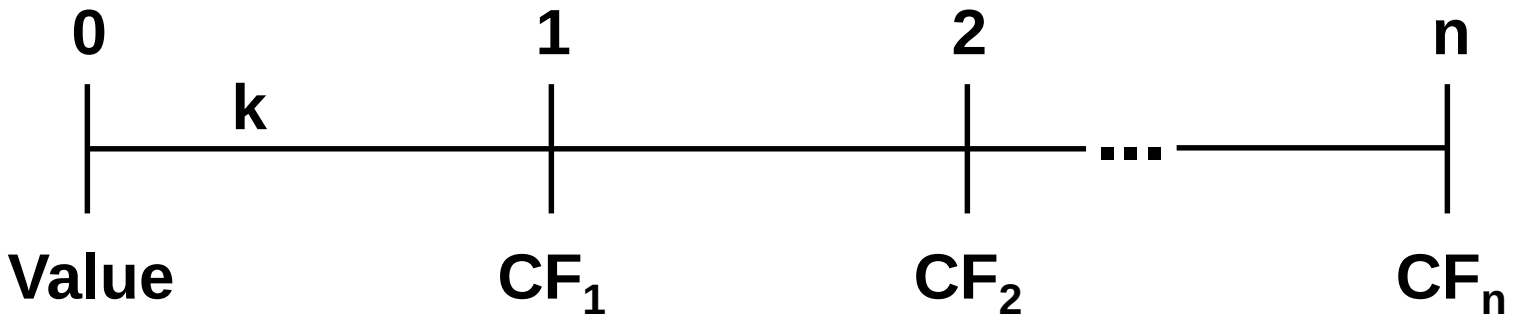


Source: http://www.investopedia.com/video/

Bond Definitions

- Par value (face value)– face amount of the bond, which is paid at maturity (assume $1,000).
- Coupon interest rate – stated interest rate (generally fixed) paid by the issuer. Multiply by par to get dollar payment of interest.
- Maturity date – years until the bond must be repaid.
- Issue date – when the bond was issued.
- Yield to maturity - rate of return earned on a bond held until maturity (also called the "promised yield").

The value of financial assets



$$\text{Value} = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_n}{(1+r)^n}$$

Present Value of Cash Flows as Rates Change

- Bond Value = PV of coupons + PV of par
- Bond Value = PV annuity + PV of lump sum

$$\text{Value} = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_n}{(1+r)^n}$$

Determinants of interest rates

The discount rate (r_d) is the opportunity cost of capital, and is the rate that could be earned on alternative investments of equal risk.

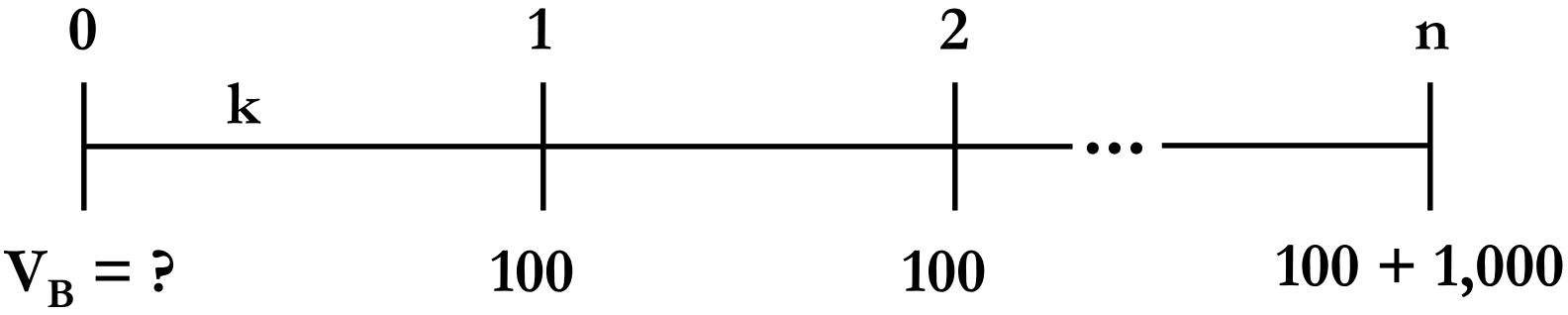
$$\mathbf{Rd = r^* + IP + DRP + LP + MRP}$$

Rd	=	required return on a debt security
r*	=	real risk-free rate of interest
IP	=	inflation premium
DRP	=	default risk premium
LP	=	liquidity premium
MRP	=	maturity risk premium

Premiums added to r* for different types of debt

	IP	MRP	DRP	LP
S-T Treasury	✓			
L-T Treasury	✓	✓		
S-T Corporate	✓		✓	✓
L-T Corporate	✓	✓	✓	✓

What is the value of a 10-year, 10% annual coupon bond, if r_d = 10%?



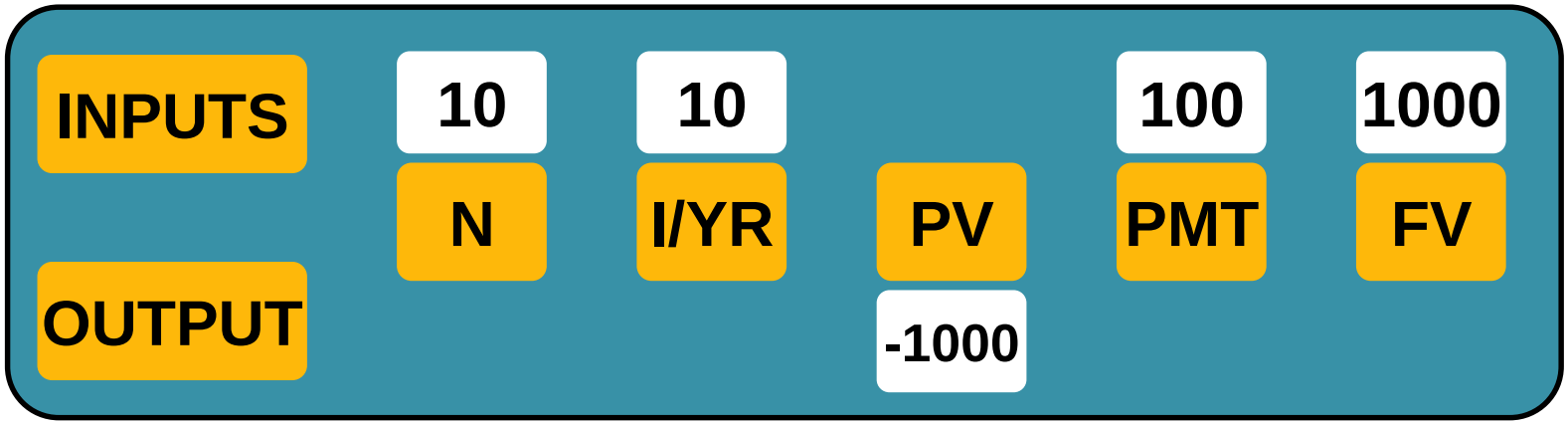
$$V_B = \frac{\$100}{(1.10)^1} + \ldots + \frac{\$100}{(1.10)^{10}} + \frac{\$1,000}{(1.10)^{10}}$$

$$V_B = \$90.91 + \ldots + \$38.55 + \$385.54$$

$$V_B = \$1,000$$

Using a financial calculator to value a bond

This bond has a $1,000 lump sum due at t = 10, and annual $100 coupon payments beginning at t = 1 and continuing through t = 10, the price of the bond can be found by solving for the PV of these cash flows.



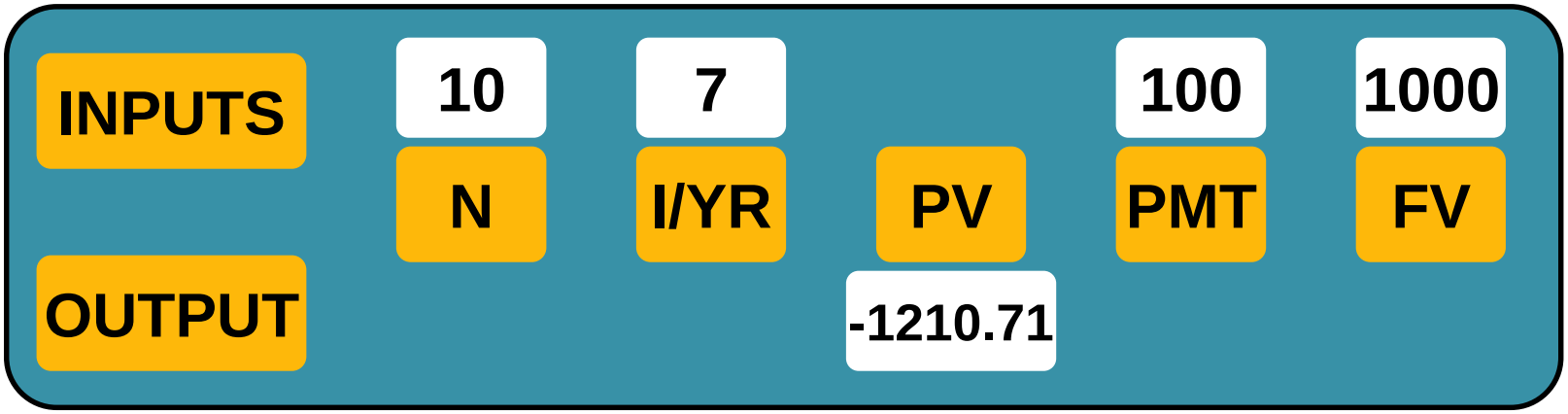
An example: Increasing inflation and r_d

Suppose inflation rises by 3%, causing r_d = 13%. When r_d rises above the coupon rate, the bond's value falls below par, and sells at a **discount**.

	N	I/YR	PV	PMT	FV
INPUTS	10	13		100	1000
OUTPUT			-837.21		

An example: Decreasing inflation and r_d

Suppose inflation falls by 3%, causing r_d = 7%. When r_d falls below the coupon rate, the bond's value rises above par, and sells at a **premium**.

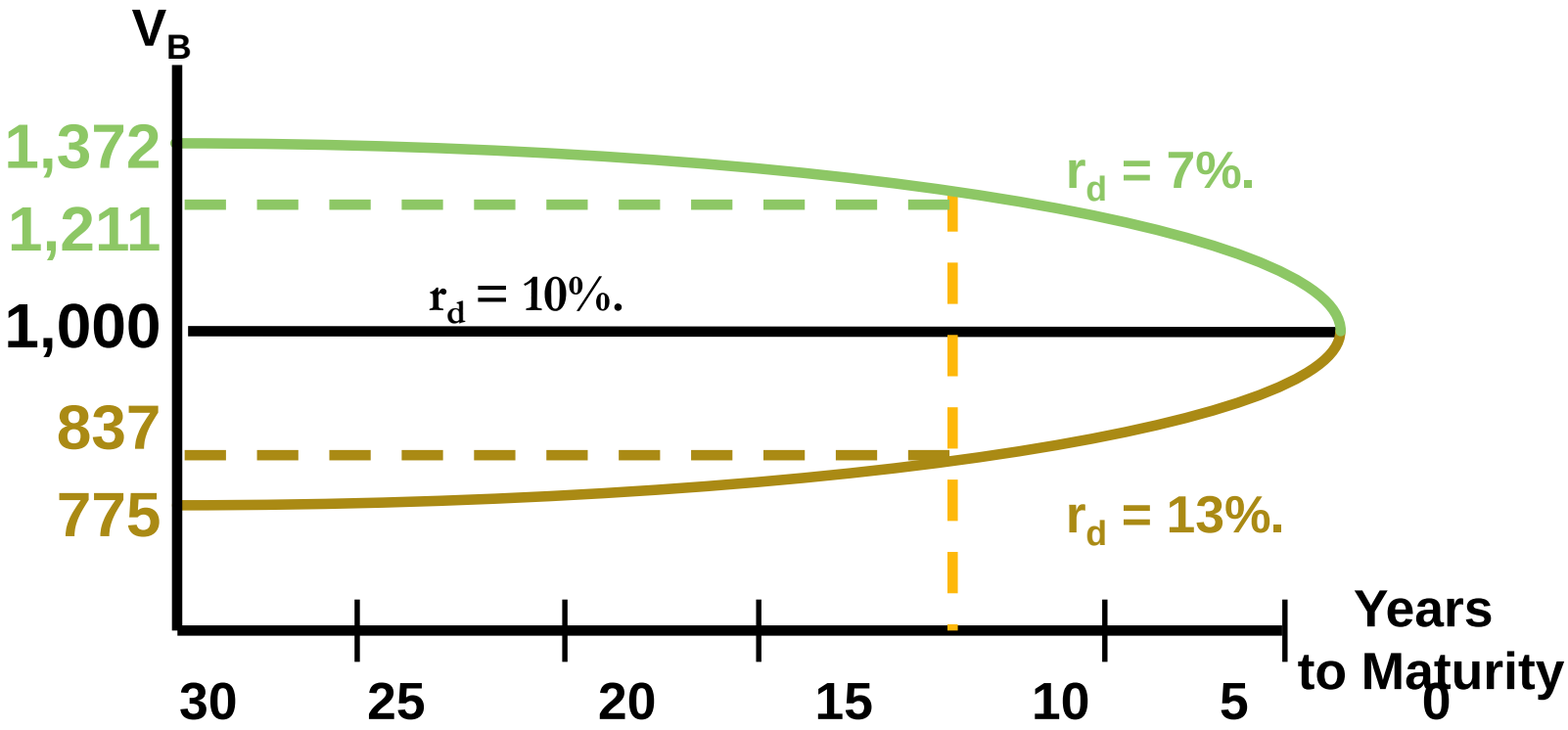


Bond values over time

- At maturity, the value of any bond must equal its par value.
- If r_d remains constant:
  - The value of a premium bond would decrease over time, until it reached $1,000.
  - The value of a discount bond would increase over time, until it reached $1,000.
  - A value of a par bond stays at $1,000.

The price path of a bond

What would happen to the value of this bond if its required rate of return remained at 10%, or at 13%, or at 7% until maturity?



Bond Prices: Relationship Between Coupon and Yield

- If coupon rate < r_d, discount.
- If coupon rate = r_d, par bond.
- If coupon rate > r_d, premium.
- If r_d rises, price falls.
- Price = par at maturity.

Computing Yield-to-maturity

- Yield-to-maturity is the rate implied by the current bond price
- Finding the YTM requires trial and error if you do not have a financial calculator and is similar to the process for finding r with an annuity
- If you have a financial calculator, enter N, PV, PMT, and FV, remembering the sign convention (PMT and FV need to have the same sign, PV the opposite sign)

What is the YTM on a 10-year, 9% annual coupon, $1,000 par value bond, selling for $887?

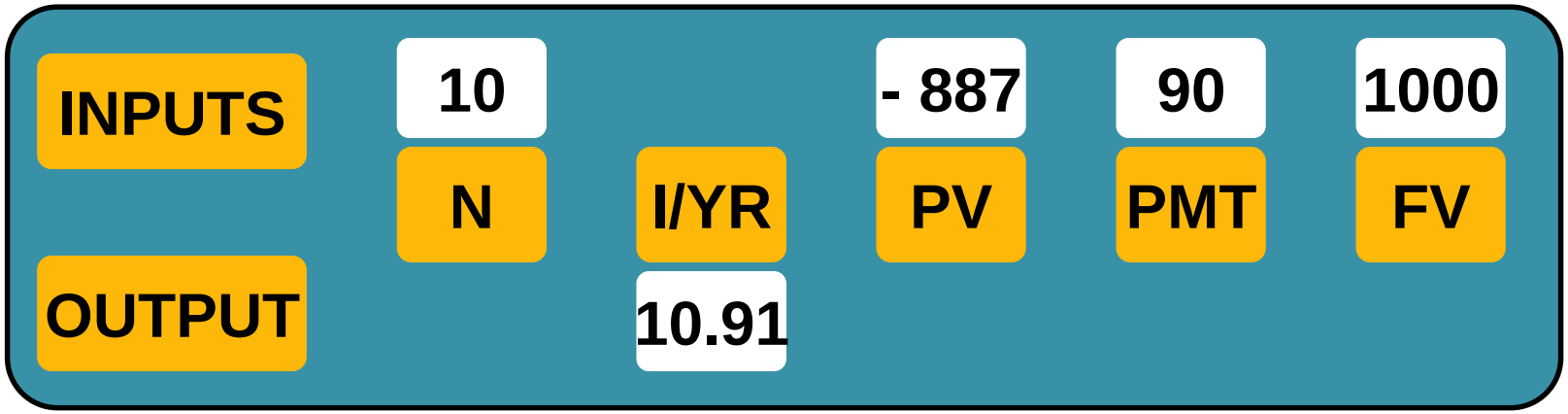
Must find the r_d that solves this model.

$$V_B = \frac{INT}{(1+r_d)^1} + \ldots + \frac{INT}{(1+r_d)^N} + \frac{M}{(1+r_d)^N}$$

$$\$887 = \frac{90}{(1+r_d)^1} + \ldots + \frac{90}{(1+r_d)^{10}} + \frac{1{,}000}{(1+r_d)^{10}}$$

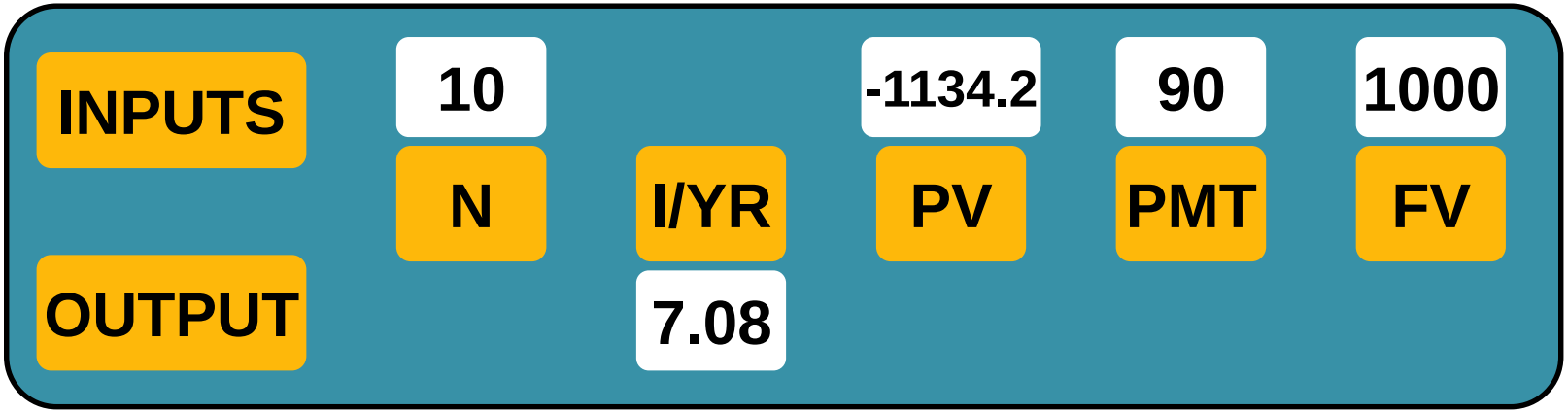
Using a financial calculator to find YTM

Solving for I/YR, the YTM of this bond is 10.91%. This bond sells at a discount, because YTM > coupon rate.



Find YTM, if the bond price was $1,134.20.

Solving for I/YR, the YTM of this bond is 7.08%. This bond sells at a premium, because YTM < coupon rate.



YTM with Annual Coupons

- Consider a bond with a 10% annual coupon rate, 15 years to maturity and a par value of $1000. The current price is $928.09.
  - Will the yield be more or less than 10%?
  - N = 15; PV = -928.09; FV = 1000; PMT = 100
  - CPT I/Y = 11%

YTM with Semiannual Coupons

- Suppose a bond with a 10% coupon rate and semiannual coupons, has a face value of $1000, 20 years to maturity and is selling for $1197.93.
  - Is the YTM more or less than 10%?
  - What is the semiannual coupon payment?
  - How many periods are there?
  - N = 40; PV = -1197.93; PMT = 50; FV = 1000; CPT I/Y = 4% (Is this the YTM?)
  - YTM = 4%*2 = 8%

Current Yield vs. Yield to Maturity

- Current Yield = annual coupon / current price
- Capital Gain Yield = change in price / beginning price

- Yield to maturity = current yield + capital gains yield

Example

- 10% coupon bond, with semiannual coupons, face value of 1000, 20 years to maturity, $1197.93 price
  - Current yield = 100 / 1197.93 = .0835 = 8.35%
  - Price in one year, assuming no change in YTM = 1193.68
  - Capital gain yield = (1193.68 – 1197.93) / 1197.93 = -.0035 = -.35%
  - YTM = 8.35 - .35 = 8%, which the same YTM computed earlier

Bond Pricing Theorems

- Bonds of similar risk (and maturity) will be priced to yield about the same return, regardless of the coupon rate
- If you know the price of one bond, you can estimate its YTM and use that to find the price of the second bond
- This is a useful concept that can be transferred to valuing assets other than bonds

Quick Quiz

- What factors determine the required return on bonds?
- How do you find the value of a bond?

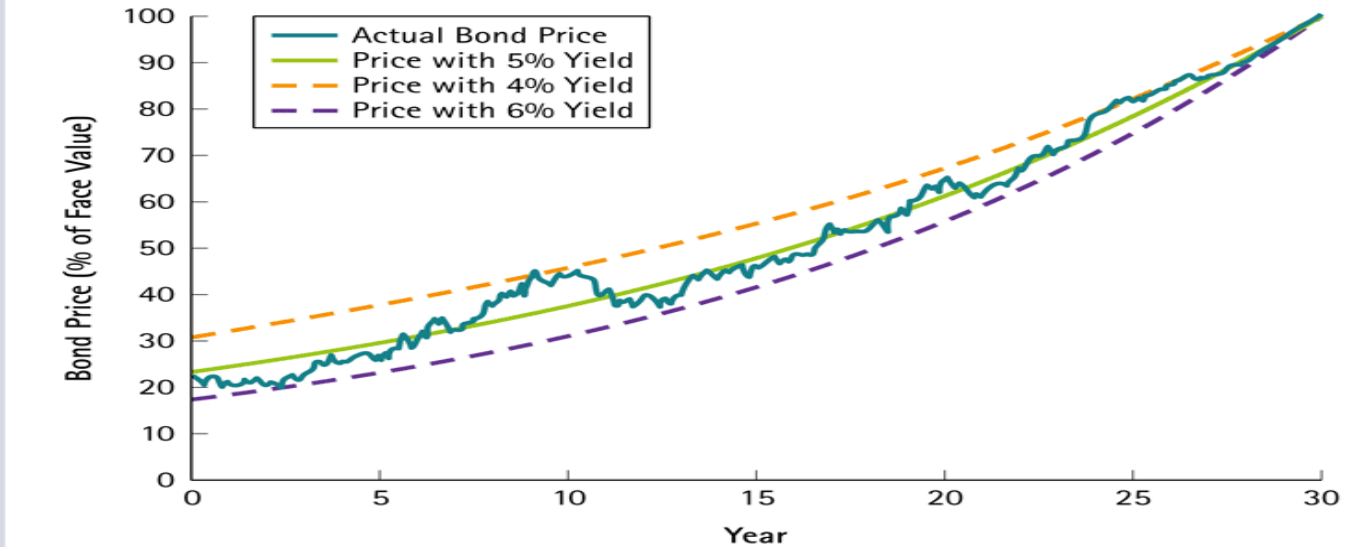
Why Bond Prices Change

- Bond Prices in Practice
  - Bond prices are subject to the effects of both passage of time and changes in interest rates.
  - Prices converge to face value due to the time effect, but move up and down because of changes in yields.

YTM and Bond Price Fluctuations over Time

Panel (a) The Bond's Yield to Maturity over Time

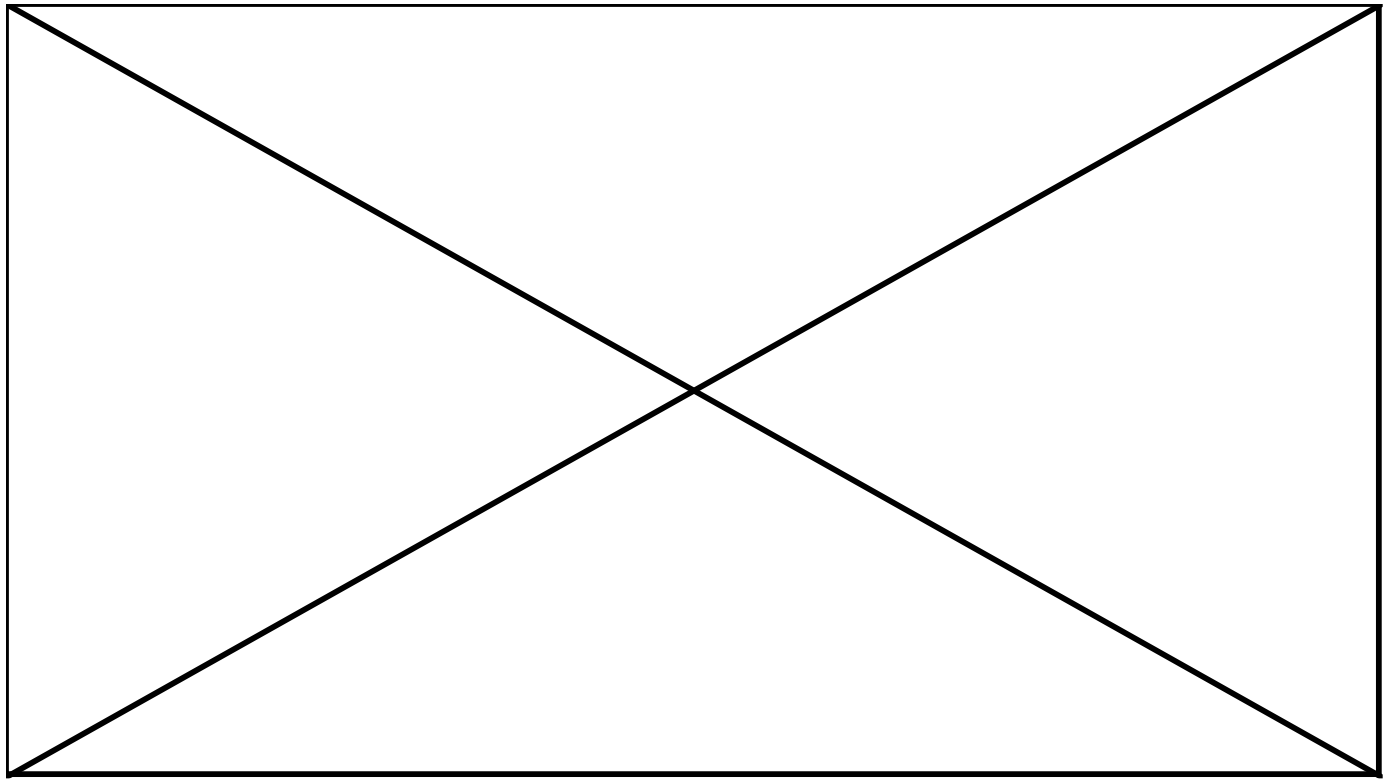
Panel (b) The Bond's Price over Time (Price = $100 on Maturity Date)



Why Bond Prices Change

- Interest Rate Risk and Bond Prices
  - Effect of time on bond prices is predictable, but unpredictable changes in rates also affect prices.
  - Bonds with different characteristics will respond differently to changes in interest rates
  - Investors view long-term bonds to be riskier than short-term bonds.

Bond Investing



Source: http://www.investopedia.com/video/

Bond Ratings

- Bond ratings help investors assess creditworthiness
- The rating depends on
  - the risk of bankruptcy
  - bondholders' claim to assets in the event of bankruptcy.
- Broad Classifications
  - Investment-grade bonds
  - Speculative bonds
    - junk bonds
    - high-yield bonds

Bond Ratings and the Number of U.S. Public Firms with those Ratings at the End of 2009

Moody's	Standard & Poor's	Number of Public Firms	Description (Moody's)
Investment Grade Debt			
Aaa	AAA	5	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged."
Aa	AA	28	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds.
A	A	164	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate at present, but may not remain that way.
Baa	BBB	399	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured).

(cont.)

Bond Ratings and the Number of U.S. Public Firms with those Ratings at the End of 2009 *(cont.)*

Speculative Bonds ("Junk Bonds")			
Ba	BB	318	Judged to have speculative elements; their future cannot be considered as well assured.
B	B	296	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments over any long period of time may be small.
Caa	CCC	27	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca	CC	4	Are speculative to a high degree. Such issues are often in default or have other marked shortcomings.
C	C, D	8	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

Source: www.moodys.com and S&P Compustat.

Features of bonds

- Call provisions
- Floating rate bond
- Zero coupon bond
- STRIPS (Separate Trading of Registered Interest and Principal Securities)

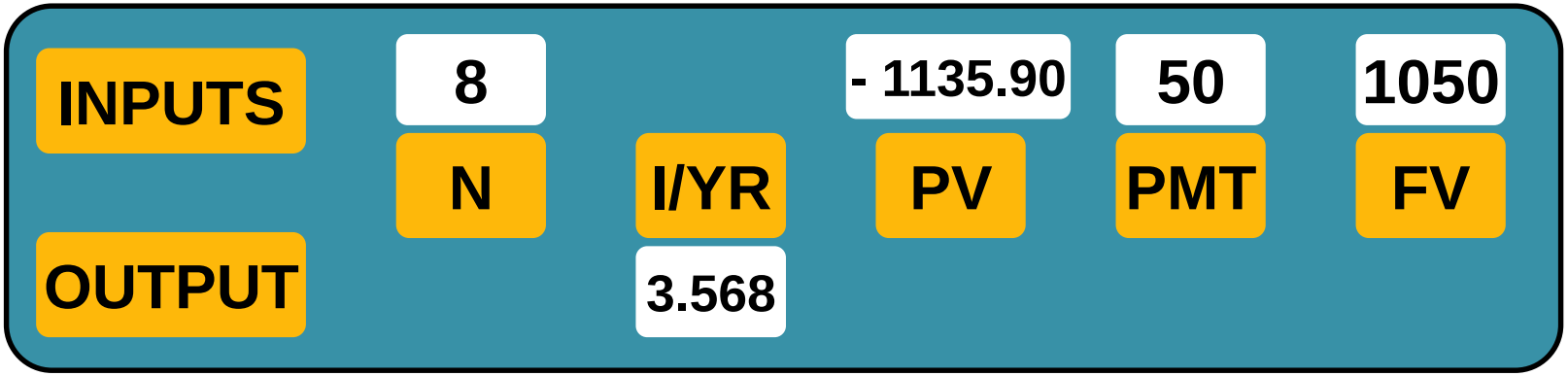
Call provisions

- Allows issuer to refund the bond issue if rates decline (helps the issuer, but hurts the investor).
- Borrowers are willing to pay more, and lenders require more, for callable bonds.
- Most bonds have a deferred call and a declining call premium.

Example

A 10-year, 10% semiannual coupon bond selling for $1,135.90 can be called in 4 years for $1,050, what is its yield to call (YTC)?

- The bond's yield to maturity can be determined to be 8%. Solving for the YTC is identical to solving for YTM, except the time to call is used for N and the call premium is FV.
- 3.568% represents the periodic semiannual yield to call.



Floating Rate Bonds

- Coupon rate floats depending on some index value
- Examples – adjustable rate mortgages and inflation-linked Treasuries
- Coupons may have a "collar" – the rate cannot go above a specified "ceiling" or below a specified "floor"

Zero-Coupon Bonds

- Make no periodic interest payments (coupon rate = 0%)
- The entire yield-to-maturity comes from the difference between the purchase price and the par value
- Cannot sell for more than par value (also known as pure discount bond)
- Although there is no interest, your compensation comes from the difference between the initial price and the face value.

Zero-Coupon Bond: YTM

- For a zero coupon bond that matures in n years, its yield, YTM_n, satisfies:

$$P = \frac{FV}{(1 + YTM_n)^n}$$

- Therefore, $$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1$$
- For example, what is the YTM of an one-year zero-coupon bond with a face value of 100 trading at the price of 96.62? (3.5%)

STRIPS

- A default-free zero-coupon bond that matures on date n provides a risk-free return over the same period. Thus, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.
- We can find risk-free interest rate (yield) with T-bills and STRIPS.
- By convention, price is quoted as a percentage of the face value.

Example

U.S. Treasury Strips

Friday, September 13, 2013

U.S. zero-coupon STRIPS allow investors to hold the interest and principal components of eligible Treasury notes and bonds as separate securities. STRIPS offer no interest payment; investors receive payment only at maturity. Quotes are as of 3 p.m. Eastern time based on transactions of $1 million or more. Yields calculated on the ask quote.

Maturity	Bid	Asked	Chg	Asked yield
Treasury Bond, Stripped Principal				
2015 Feb 15	99.581	99.595	0.004	0.29
2015 Aug 15	99.158	99.177	unch.	0.43
2015 Nov 15	98.862	98.883	-0.017	0.52

- How is the asked yield calculated?
- Days between Sept 13, 2013 and Feb 15, 2015 = 520
- $(1+y)^{520/365} = \frac{100}{99.595}, y = 0.002853$

Other types (features) of bonds

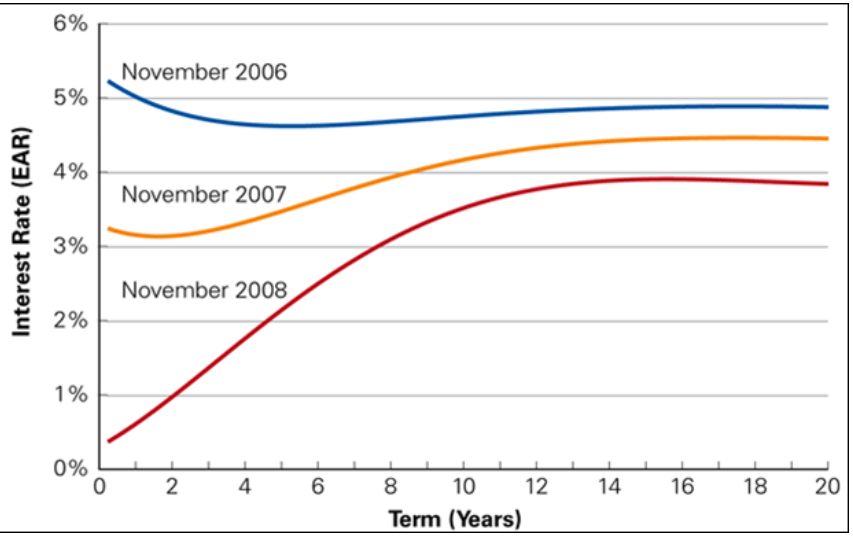
- Convertible bond – may be exchanged for common stock of the firm, at the holder's option.
- Putable bond – allows holder to sell the bond back to the company prior to maturity.
- Income bond – pays interest only when interest is earned by the firm.
- Indexed bond – interest rate paid is based upon the rate of inflation.

Term Structure of Interest Rate

- The term structure of interest rates (also called zero-coupon yield curve) plots the interest rate (yield on discount bond) at which you can borrow/lend as a function of the borrowing/lending horizon (time to maturity)

Term Structure of Interest Rates, Nov. 2006, 2007, 2008 (Based on Yields of U.S. Treasury STRIPS)

Term (years)	Date Nov-06	Nov-07	Nov-08
0.5	5.15%	3.20%	0.44%
1	5.02%	3.15%	0.60%
2	4.83%	3.14%	0.96%
3	4.71%	3.20%	1.35%
4	4.64%	3.32%	1.75%
5	4.62%	3.47%	2.13%
6	4.62%	3.63%	2.49%
7	4.65%	3.78%	2.81%
8	4.68%	3.93%	3.09%
9	4.71%	4.06%	3.32%
10	4.75%	4.17%	3.51%
15	4.87%	4.44%	3.90%
20	4.88%	4.45%	3.84%



Important Note:

- We've used YTM, a discount rate, that equates the PV of the promised bond payments to the current market price of the bond.
- The plot of the yields of coupon bonds of different maturities is called the coupon-paying yield curve. The "yield curve" often refers to the coupon-paying Treasury yield curve. Strictly speaking, the yield on a coupon bond does not correspond to a discount rate at which you can borrow/lend at any horizon and is not useful for discounting!

Valuing a Coupon Bond Using Zero-Coupon Yields

- Using the Law of One Price and the yields of default-free zero-coupon bonds, we can determine the price and yield of any other default-free bond. The price of an annual-pay coupon bond is given by:

$$P_T = \sum_{j=1}^{T} \frac{C}{(1+r_j)^j} + \frac{FV}{(1+r_T)^T}$$

- where C is the annual coupon payment, FV is the face value of the bond, and rj is the j-period interest rate. These interest rates are also yields on discount bonds; thus, they can be obtained from the prices of discount bonds

Example

- Given the following prices of STRIPS, what is the price for a 5-year annual-pay coupon bond with a coupon rate of 10% and a face value of $1000?

Years to Maturity	1	2	3	4	5
STRIPS Price	98	95	92	89	85

$$P = 100 \times 98\% + 100 \times 95\% + 100 \times 92\% + 100 \times 89\% + 1100 \times 85\% = 1309$$

Quick Quiz

- Why do bond prices change?
- What are bond ratings, and why are they important?
- What is a bond feature that benefits bond issuers?
- What are the differences between valuing a bond using YTM and "yield curve"?

Bond Characteristics and Interest Rate Risk

Bond Characteristic	Effect on Interest Rate Risk
Longer term to maturity	Increase
Higher coupon payments	Decrease

In-Class Activities

- The Interest Rate Sensitivity of Bonds
- Coupons and Interest Rate Sensitivity

The Interest Rate Sensitivity of Bonds

Problem:

- Consider a 10-year coupon bond and a 30-year coupon bond, both with 10% annual coupons.
- By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%?

The Interest Rate Sensitivity of Bonds

Solution

Plan:

- We need to compute the price of each bond for each yield to maturity and then calculate the percentage change in the prices.
- For both bonds, the cash flows are $10 per year for $100 in face value and then the $100 face value repaid at maturity.
- The only difference is the maturity: 10 years and 30 years.

The Interest Rate Sensitivity of Bonds

Execute:

Insert Table portion of Example 6.8 page 160.

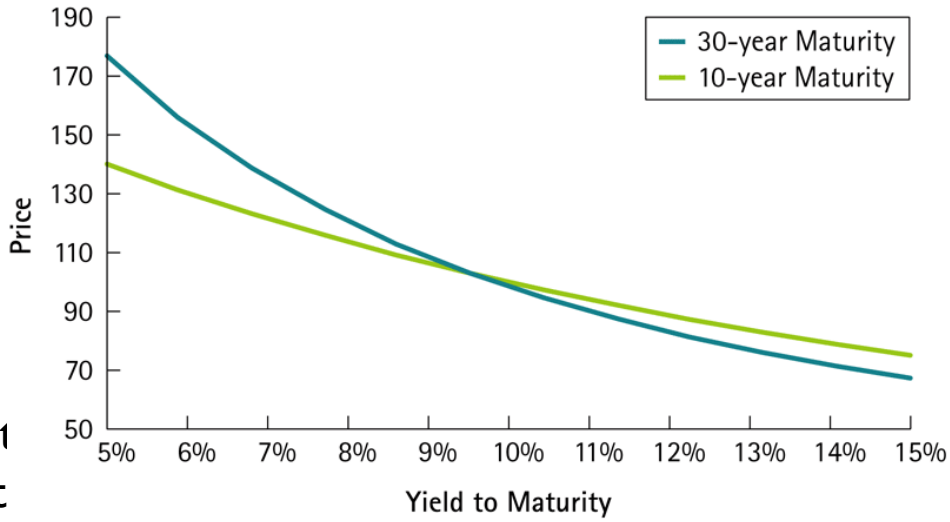
YTM	10-Year, 10% Annual Coupon Bond	30-Year, 10% Annual Coupon Bond
5%	$10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{10}}\right) + \frac{100}{1.05^{10}} = \$138.61$	$10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$
6%	$10 \times \frac{1}{0.06}\left(1 - \frac{1}{1.06^{10}}\right) + \frac{100}{1.06^{10}} = \$129.44$	$10 \times \frac{1}{0.06}\left(1 - \frac{1}{1.06^{30}}\right) + \frac{100}{1.06^{30}} = \$155.06$

The price of the 10-year bond changes by (129.44 - 138.61) / 138.61 = -6.6% if its yield to maturity increases from 5% to 6%.
For the 30-year bond, the price change is (155.06 - 176.86) / 176.86 = -12.3%.

The Interest Rate Sensitivity of Bonds

Evaluate:

- The 30-year bond is twice as sensitive to a change in the yield than is the 10-year bond.
- In fact, if we graph the price and yields of the two bonds, we can see that the line for the 30-year bond, shown in blue, is steeper throughout than the green line for the 10-year bond, reflecting it heightened sensitivity to interest rate changes.



Coupons and Interest Rate Sensitivity

Problem:

- Consider two bonds, each pays semi-annual coupons and 5 years left until maturity.
- One has a coupon rate of 5% and the other has a coupon rate of 10%, but both currently have a yield to maturity of 8%.
- How much will the price of each bond change if its yield to maturity decreases from 8% to 7%?

Coupons and Interest Rate Sensitivity

Solution

Plan:

- As in Example 6.8, we need to compute the price of each bond at 8% and 7% yield to maturities and then compute the percentage change in price.
- Each bond has 10 semi-annual coupon payments remaining along with the repayment of par value at maturity.
- The cash flows per $100 of face value for the first bond are $2.50 every 6 months and then $100 at maturity.

Coupons and Interest Rate Sensitivity

Solution

Plan (cont'd):

- The cash flows per $100 of face value for the second bond are $5 every 6 months and then $100 at maturity.
- Since the cash flows are semi-annual, the yield to maturity is quoted as a semi-annually compounded APR, so we convert the yields to match the frequency of the cash flows by dividing by 2.
- With semi-annual rates of 4% and 3.5%, we can use Eq.(6.3) to compute the prices.

Coupons and Interest Rate Sensitivity

Execute:

- The 5% coupon bond's price changed from $87.83 to $91.68, or 4.4%, but the 10% coupon bond's price changed from $108.11 to $112.47, or 4.0%.
- You can calculate the price change very quickly with a financial calculator. Taking the 5% coupon bond for example:

Given:	10	4		2.50	100
Solve for:			**-87.83**		
Excel Formula: =PV(RATE,NPER,PMT,FV)=PV(.04,10,2.5,100)					

YTM	5-Year, 5% Coupon Bond	5-Year, 10% Coupon Bond
8%	$2.50 \times \frac{1}{0.04}\left(1 - \frac{1}{1.04^{10}}\right) + \frac{100}{1.04^{10}} = \$87.83$	$5 \times \frac{1}{0.04}\left(1 - \frac{1}{1.04^{10}}\right) + \frac{100}{1.04^{10}} = \$108.11$
7%	$2.50 \times \frac{1}{0.035}\left(1 - \frac{1}{1.035^{10}}\right) + \frac{100}{1.035^{10}} = \$91.68$	$5 \times \frac{1}{0.035}\left(1 - \frac{1}{1.035^{10}}\right) + \frac{100}{1.035^{10}} = \$112.47$

Coupons and Interest Rate Sensitivity

Evaluate:

- The bond with the smaller coupon payments is more sensitive to changes in interest rates.
- Because its coupons are smaller relative to its par value, a larger fraction of its cash flows are received later.
- As we learned in Example 6.8, later cash flows are affected more greatly by changes in interest rates, so compared to the 10% coupon bond, the effect of the interest change is greater for the cash flows of the 5% bond.

End of Lesson